Limits And Continuity

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Limits and Continuity

Subtopics: Properties of Integrals, Fundamental Theorem of Calculus (First), Concavity, Tangents To Curves, Mean Value Theorem, Continuities and Discontinuities, Derivative Tables

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 6

x	−1.5	−1.0	−0.5	0	0.5	1.0	1.5
$f(x)$	−1	−4	−6	−7	−6	−4	−1
$f'(x)$	−7	−5	−3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\, dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0 \end{cases}$.

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

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Question 2

Qualification: AP Calculus AB

Areas: Limits and Continuity, Integration, Applications of Integration

Subtopics: Continuities and Discontinuities, Calculating Limits Algebraically, Average Value of a Function, Properties of Integrals, Integration Technique – Standard Functions, Differentiability

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Very Hard / Question Number: 6

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

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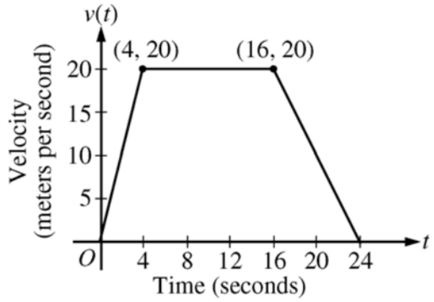
Question 3

Qualification: AP Calculus AB

Areas: Integration, Applications of Integration, Limits and Continuity, Applications of Differentiation

Subtopics: Interpreting Meaning in Applied Contexts, Kinematics (Displacement, Velocity, and Acceleration), Integration Technique – Geometric Areas, Differentiability, Derivative Graphs, Rates of Change (Average), Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Hard / Question Number: 5



5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

Question 4

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration

Subtopics: Interpreting Meaning in Applied Contexts, Derivative Tables, Kinematics (Displacement, Velocity, and Acceleration), Riemann Sums – Trapezoidal Rule, Intermediate Value Theorem, Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2006-Form-B / Difficulty: Hard / Question Number: 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft / sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\,dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\,dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\,dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

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Question 5

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Integration

Subtopics: Intermediate Value Theorem, Mean Value Theorem, Fundamental Theorem of Calculus (Second), Tangents To Curves, Derivative Tables

Paper: Part A-Calc / Series: 2007 / Difficulty: Very Hard / Question Number: 3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

   (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\, dt$. Find the value of $w'(3)$.

   (d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

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Question 6

Qualification: AP Calculus AB

Areas: Differential Equations, Limits and Continuity

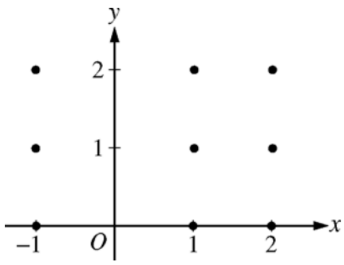
Subtopics: Sketching Slope Field, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Standard Functions, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule, Points Of Inflection, Calculating Limits Algebraically, Tangents To Curves

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 6

6. Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

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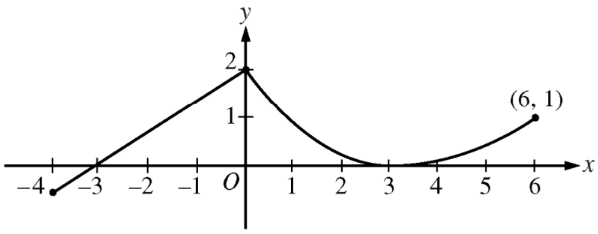
Question 8

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Differentiability, Rates of Change (Average), Mean Value Theorem, Concavity, Fundamental Theorem of Calculus (Second)

Paper: Part A-Calc / Series: 2009-Form-B / Difficulty: Somewhat Challenging / Question Number: 3



Graph of f

3. A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

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Question 9

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration, Differentiation

Subtopics: Continuities and Discontinuities, Average Value of a Function, Integration Technique – Exponentials, Integration Technique – Trigonometry, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Somewhat Challenging / Question Number: 6

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

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Question 10

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration

Subtopics: Continuities and Discontinuities, Average Value of a Function, Interpreting Meaning in Applied Contexts, Modelling Situations, Calculating Limits Algebraically, Accumulation of Change

Paper: Part A-Calc / Series: 2011-Form-B / Difficulty: Easy / Question Number: 2

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

Question 11

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Differentiation, Integration

Subtopics: Differentiation Technique – Chain Rule, Differentiation Technique – Standard Functions, Tangents To Curves, Continuities and Discontinuities, Integration Technique – Substitution

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Medium / Question Number: 4

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\,dx$.

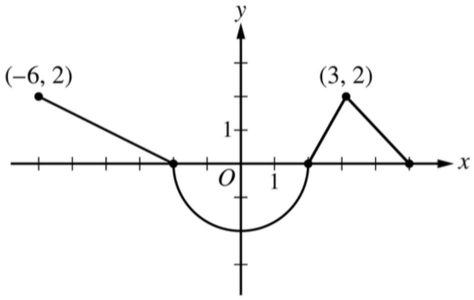
Question 12

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Integration Technique – Geometric Areas, Derivative Graphs, Increasing/Decreasing , Global or Absolute Minima and Maxima, Differentiability

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 3



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

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Question 13

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Rates of Change (Average), Tangents To Curves, Global or Absolute Minima and Maxima, L'Hôpital's Rule, Calculating Limits Algebraically, Differentiation Technique – Product Rule, Differentiation Technique – Exponentials, Differentiation Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Somewhat Challenging / Question Number: 5

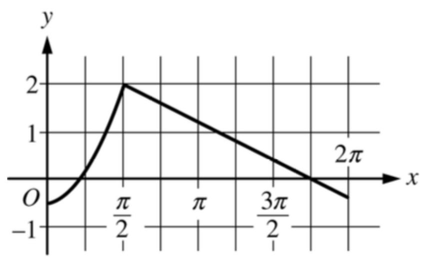
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

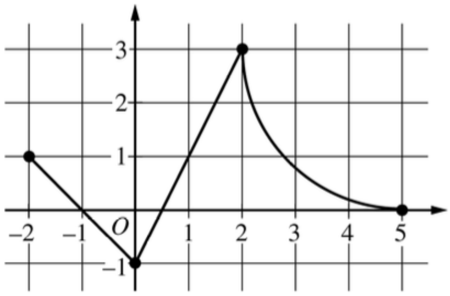
Question 14

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (First), Global or Absolute Minima and Maxima, Calculating Limits Algebraically, Integration Graphs

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 3



Graph of f

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3-\sqrt{5})$ is on the graph of f.

(a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

(b) Evaluate $\int_{3}^{5} (2f'(x)+4)\,dx$.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

(d) Find $\lim\limits_{x\to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

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Question 15

Qualification: AP Calculus AB

Areas: Limits and Continuity, Differentiation

Subtopics: Differentiation Technique – Product Rule, L'Hôpital's Rule, Calculating Limits Algebraically, Differentiation Technique – Chain Rule, Continuities and Discontinuities

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Very Hard / Question Number: 6

6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

(a) Find $h'(2)$.

(b) Let a be the function given by $a(x) = 3x^3h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

(c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

(d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

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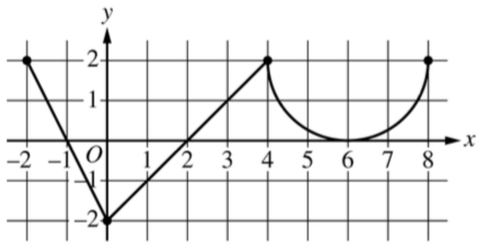
Question 16

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Concavity, Derivative Graphs, Global or Absolute Minima and Maxima, Integration Technique – Geometric Areas, L'Hôpital's Rule, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Medium / Question Number: 4



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

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